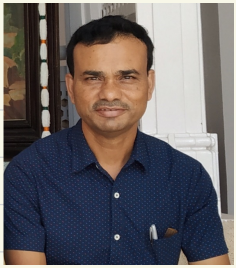

RKBhuyan
Asso. professor
Tihu College

Reduction

The action of making Something smaller or less amount, degree, or size.

$\boxed{\text{Reduction Formulae :-}}$

We know that

1. $\int \sin x\,dx = -\cos x$

2. $\int \sin^2 x\,dx = \frac{1}{2}\int 2\sin^2 x\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x = 1 - \cos 2x$

$$= \frac{1}{2}\int (1 - \cos 2x)\,dx$$

$$= \frac{1}{2}x - \frac{\sin 2x}{4}$$

3. $\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx$

$\frac{d}{dx}(\cos x) = -\sin x$

$\Rightarrow d(\cos x) = -\sin x\,dx$

$$= -\int (1 - \cos^2 x)\,d(\cos x)$$

$$= -\int d(\cos x) + \int \cos^2 x\,d(\cos x)$$

$$= -\cos x + \frac{\cos^3 x}{3}$$

$$\left[\, d(\cos x) = \frac{d(\cos x)}{dx}dx = -\sin x\, dx \,\right]$$

HW

$$\int \sin^4 x\, dx = \int (\sin^2 x)^2 dx$$
$$= \frac{1}{4}\int (2\sin^2 x)^2 dx$$
$$= \frac{1}{4}\int (1-\cos 2x)^2 dx$$

$$\int \sin mx\, dx = -\frac{\cos mx}{m}$$

$$\int \sin^5 x\, dx$$

$$= \int \sin^4 x \, \sin x\, dx$$
$$= -\int (1-\cos^2 x)^2 d(\cos x), \quad t = \cos x$$
$$= -\int (1-t^2)^2 dt$$
$$= -\int (1-2t^2+t^4)\, dt = -$$
$$= -t + 2\frac{t^3}{3} - \frac{t^5}{5}$$
$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x \quad \#$$

Reduction formula for $\int \sin^n x\,dx$

$$\int \sin^n x\,dx$$

$$\int uv\,dx = u\int v\,dx - \int\left\{\frac{d}{dx}(u)\int v\,dx\right\}dx$$

$$= \int \sin^{n-1}x \cdot \sin x\,dx$$

$$= \sin^{n-1}x \int \sin x\,dx - \int\left\{\frac{d}{dx}(\sin^{n-1}x)\int \sin x\,dx\right\}dx$$

Using the formula of Integration by Parts

$$= -\sin^{n-1}x\cos x - \int (n-1)\sin^{n-2}x \cdot \cos x(-\cos x)\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,(1-\sin^2 x)\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx - (n-1)\int \sin^n x\,dx$$

$$\therefore (1+n-1)\int \sin^n x\,dx = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx$$

$$\therefore \int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

This is the required Reduction Formulae.

Note that

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n}\int_0^{\pi/2} \sin^{n-2} x\, dx$$

Since $\left[\dfrac{\sin^{n-1}x \cos x}{n}\right]_0^{\pi/2} = 0$

$$= \begin{cases} \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\dfrac{3}{4}\cdot\dfrac{1}{2}\displaystyle\int_0^{\pi/2} dx & \text{when } n \text{ is even} \\ \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdots\dfrac{4}{5}\cdot\dfrac{2}{3}\cdot\displaystyle\int_0^{\pi/2} \sin x\, dx & \text{when } n \text{ is odd.} \end{cases}$$

$$= \begin{cases} \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdots\dfrac{3}{4}\cdot\dfrac{1}{2}\cdot\dfrac{\pi}{2} & \text{when } n \text{ is even} \\ \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdots\dfrac{4}{5}\cdot\dfrac{2}{3} & \text{when } n \text{ is odd} \end{cases}$$

This is Walli's formula.

$$\int_0^{\pi/2} dx = x\Big]_0^{\pi/2} = \frac{\pi}{2}$$

$$\int_0^{\pi/2} \sin x\, dx = -\cos x\Big]_0^{\pi/2} = -[0-1] = 1$$

RKBhuyan
Tihu College

Find $\int \cos x \, dx$

~~Home work~~

$$\int \cos^2 x \, dx$$

$$\int \cos^3 x \, dx$$

$\vdots$

~~Home work~~

?

Reduction formulae for $\int \cos^n x \, dx$.

HW Find Walli's formula for $\int_0^{\pi/2} \cos^n x \, dx$

RKBhuyan
Tihu College

Que.:

3.4. If n is a positive integer, then prove that

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$$

$$= \begin{cases} \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\cdots\dfrac{3}{4}\cdot\dfrac{1}{2}\cdot\dfrac{\pi}{2}, & \text{if } n \text{ is even};\\[2ex] \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\cdots\dfrac{4}{5}\cdot\dfrac{2}{3}\cdot 1, & \text{if } n \text{ is odd}. \end{cases}$$

Reduction formulae for $\int \tan^n x\,dx$

We know that

$$\int \tan^n x\,dx = \int \tan^{n-2} x \cdot \tan^2 x\,dx$$

$1+\tan^2\theta = \sec^2\theta$

$$= \int \tan^{n-2} x\,(\sec^2 x - 1)\,dx$$

$\sec^2 x\,dx = d(\tan x)$

$$= \int \tan^{n-2} x\; d(\tan x) - \int \tan^{n-2} x\,dx$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$$

$$\left[\sec^2 x\,dx = d(\tan x)\right]$$

$$\boxed{I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}}$$

If we put $I_n = \int \tan^n x\,dx$

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$$d(\tan x) = \frac{d}{dx}(\tan x)\,dx = \sec^2 x\,dx$$

$$z = \tan x$$

$$\int \tan^{n-2} x \cdot d(\tan x) = \int z^{n-2}\,dz = \frac{z^{n-2+1}}{n-2+1} = \frac{z^{n-1}}{n-1} = \frac{\tan^{n-1} x}{n-1}$$

Que:- If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then show that $I_n + I_{n-2} = \frac{1}{n-1}$ and hence find the value of I_5.

Ans:- From the above problem we get

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

$$\Rightarrow \int_0^{\pi/4} \tan^n x \, dx = \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/4} - \int_0^{\pi/4} \tan^{n-2} x \, dx$$

$$\Rightarrow I_n = \left[\frac{1}{n-1} - 0\right] - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1} \quad \#$$

$I_5 = ?$ (HW)

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2nd Part

$$I_5 = \int_0^{\pi/4} \tan^5 x \, dx$$

We get $I_n + I_{n-2} = \frac{1}{n-1}$

$$\Rightarrow I_5 + I_3 = \frac{1}{5-1} = \frac{1}{4}$$

$$I_3 + I_1 = \frac{1}{3-1} = \frac{1}{2}$$

$I_1 = \int_0^{\pi/4} \tan x \, dx = \left[\log(\sec x)\right]_0^{\pi/4}$

$= \log\sqrt{2} - 0$

$= \frac{1}{2}\log 2$

$$I_5 = \frac{1}{4} - I_3$$

$$= \frac{1}{4} - \left(\frac{1}{2} - I_1\right)$$

$$= \frac{1}{4} - \frac{1}{2} + \frac{1}{2}\log 2$$

$$= -\frac{1}{4} + \frac{1}{2}\log 2$$

HW find I_6, I_7 with the help of above Identity.