Mathematics. :: Calculus

কলন গণিত

RKBhuyan, Tihu College

Rama Kanta Bhuyan
Dept. Of Mathematics
Tihu College

bkbhuyan

গণিতৰ আদিপাঠ

Symbols :-

α — alpha	Φ — Phi
β — Beta	Ψ — Psi
γ — Gamma	π — Pi
Δ, δ — delta	λ — Lemda
ε — epsilon	μ — Mu
	ξ — Xi

π, e are irrational numbers.

$\pi = 3.1416\ldots$, $2 < e < 3$

Some Sets of Numbers:

N – The set of natural numbers
স্বাভাবিক সংখ্যার সংহতি

$= \{1, 2, 3, \ldots\ldots\}$

I or Z – The set of integers
সকল সংখ্যার সংহতি

$= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Q = The set Rational numbers.
মূলদীয় সংখ্যার সংহতি

$\left\{\frac{p}{q}, (q \neq 0), p, q \in I\right\}$

R – The set of real numbers.
বাস্তব সংখ্যার সংহতি

(মূলদীয় ∪ অমূলদীয়) সংখ্যার সংহতি

Imp. to Remember

Let $A = \{x, y, z\}$, $B = \{a, b\}$

$C = \{x, y, z, a, b\}$

$\in$ — "belongs to"

$x \in A$ but $x \notin B$

$\subset$ — Subset of

$A \subset C$, $B \subset C$

but $B \not\subset A$

$x \in A$, $A \subset C$

x → elements; A → Set; A, C → Set.

$\frac{0}{0}, \frac{a}{0}$ $(a \neq 0)$ are meaningless

— অৰ্থহীন

— does not exits.

— স্থিৰ নহয় ।

∞ — is not a numbers

— $\notin R$

ই এটা নিৰ্দিষ্ট সংখ্যা অৰ্থ ৰাশি অসীমক বোজা বুজায় ।

$$|x| = x, \quad \text{if } x \geq 0$$
$$= -x, \quad \text{if } x < 0$$

মাপাংক মান

RKBhuyan

Properties

RKBhuyan

1. $|a| \geq 0$

2. $|a| = |-a|$

3. $|a+b| \leq |a| + |b|$

4. $|a|^2 = a^2$

Meaning of $|x-a| < \delta$ is

$$a - \delta < x < a + \delta$$

ব্যাখ্যা :—

$$|x-a| = x-a \quad \text{if } x-a > 0$$
$$\text{i.e. } x > a$$

$$= -(x-a) \quad \text{if } x-a < 0$$
$$= a-x \quad \text{if } x < a$$

$\therefore \quad |x-a| < \delta$

$\Rightarrow \quad x-a < \delta \quad$ if $x > a$

$\Rightarrow \quad x < a+\delta \longrightarrow (i)$

& $\quad |x-a| < \delta$

$\Rightarrow \quad a-x < \delta \quad$ if $a > x$

$\Rightarrow \quad a-\delta < x \longrightarrow (ii)$

$\therefore$ (i) and (ii) $\Rightarrow a-\delta < x < a+\delta$ #

$|x| \leq 1 \Rightarrow -1 \leq x \leq 1$

[Putting $a=0, \delta=1$ in above]

Class no 2
গণিতৰ আদিপাঠ

RKBhuyan
Tihu College

Quantities are of two types :—

1. **Constant** : ধ্রুবক রাশি
2. **Variable** : চলক "

Constant are those who takes same values in different cases.
and Variable take different value in different cases.

Constant are of two types

a. **Numeric constant**

2, −5, 3/2, 0, π ... etc.

b. **Arbitrary Constant.**

a, b, c, d, .. English alphabet from 1st.

Variables are also of two types.

1. Independent variable

স্বাধীন চলক

2. Dependent variable.

পরাধীন চলক

Let $y = 2x + 1$.

y → Dependent

x → Independent

x	y
0	1
1	3
2	9
,	,
,	,
,	,

$x, y, z, r, s, t, \ldots$ alphabets from the end.

are written as variables.

RKBhuyan

তোমালোকে। YouTube ত গৈ. MathWith RKBhuyan লিখি চার্চ কৰি দিয়া । Chanel টো পালে , যদি Subscribe কৰা Class XI-XII ৰ Mathematics ৰ বহুত Class তাত পাবা , Subscribe কৰিবলৈ তুমাক এটা e-mail address লাগিব আৰু সেইটোৰে তুমি YouTube ত login কৰি লব লাগিব ।

নাইবা

তলত দিয়া লিংকটোৰেও তুমি Channel টো পাব পাৰা

https://youtu.be/ilrbqpAsnhc

Interval: অন্তরাল

Closed interval : বন্ধ অন্তরাল

$[a,b]$; $x \in [a,b] \Rightarrow a \le x \le b$

$]a,b[$ or (a,b) : $a < x < b$.

Semi = demi = hemi = $\frac{1}{2}$

Semi Closed/Open interval

$[a,b)$. $a \le x < b$

$]a,b]$ or $(a,b]$

: $a < x \le b$.

Solution of: $(x-a)(x-b) > 0$

: $(x-a)(x-b) < 0$

If $mn > 0 \Rightarrow$

Either $m > 0$ & $n > 0$

OR $m < 0$ & $n < 0$

If $mn < 0$ then

Either: $m < 0$ and $n > 0$

OR: $m > 0$ and $n < 0$

RKBhuyan

RKBhuyan

Symbols:

$\exists$ — there exist

$\forall$ — for all

i.e. → that is

e.g. — for example

+ive or +ve — positive.

−ve or −ive — negative.

$\Rightarrow$ implied by

$\Leftrightarrow$ if and only if or iff

$\rightarrow$ tends to

$x \to 2$

Now we are ready to discuss about Function.

Function :- Let x and y be two variables. Now if x and y are so related that "for every value of x, there is a unique value of y" Then y is called a function of x and it is written as

$$y = f(x)$$

You can write it as $h(x)$, $g(x)$, $\phi(x)$ ..... etc.

Note that function is a certain type of relation.

RKBhuyan
Tihu College

Class no 3

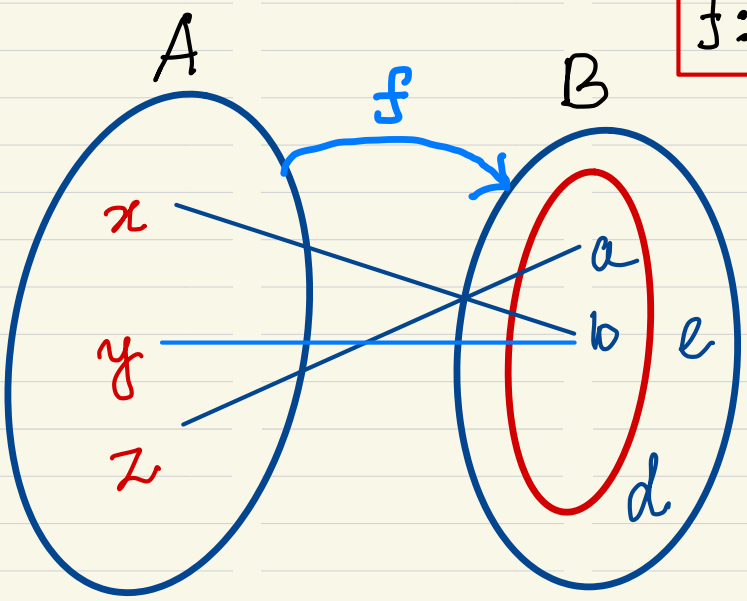
ফলন
Function

ফলন। ভাগ - ১

RKBhuyan

Function : ফলন

$f: A \to B$



Range

পৰিসৰ

image

Domain

ডমেইন

pre-image

Co-domain

কডমেইন

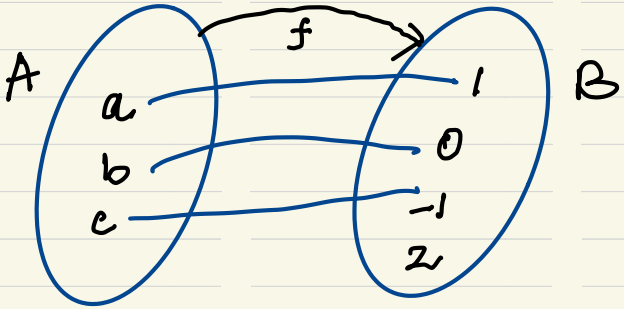
Conditions:

1. Every elt. of A must have an image in B.
2. Each elt of A has a unique image in B.

Function is also called mapping. (ম্যাপিং)

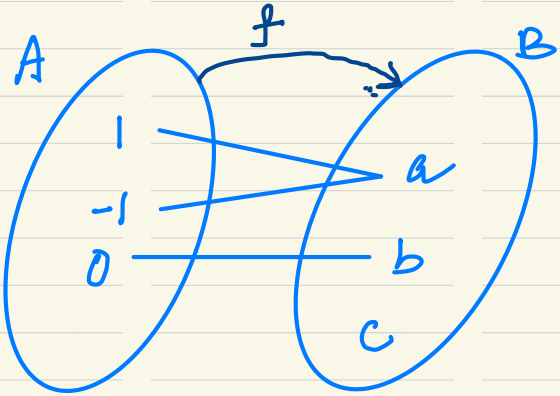
Some Interesting Function/mappings:

One – One mapping : একেকী ফলন



$f : A \to B$ is one-one (একেকী) mapping

Many – One mapping :



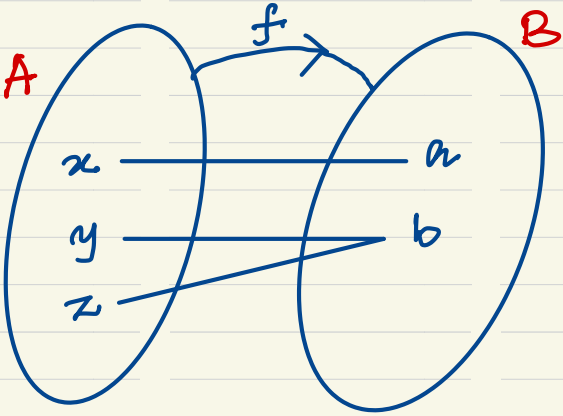
RKBhuyan

Onto Mapping

RKBhuyan

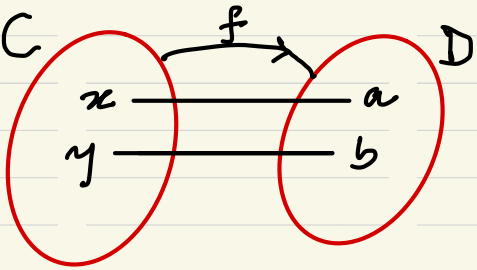
আচ্ছদক ফলন

Range = Co-domain



$f: A \to B$ is onto function.

One-One function :-



$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

and $f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$

Que:- Show that

$f(x) = 2x^2 + 3$ is one-one

Ans:- Let $x_1, x_2 \in D_f \subset R$.

Now $f(x_1) = f(x_2)$

$\Rightarrow 2x_1^2 + 3 = 2x_2^2 + 3$

$\Rightarrow 2x_1^2 = 2x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$ and $x_1 = -x_2$

$\therefore f(x_1) = f(-x_1)$

$\Rightarrow f$ is not one-one.

Onto

RKBhuyan

Que:- Let $f: \mathbb{R} \to \mathbb{R}$ be define by $f(x) = x+1$. Determine f is onto.

Ans:- Here $f(x) = x+1$, $\forall\, x \in \mathbb{R}$.

For any $y \in \mathbb{R}$ and $y = f(x)$

$$\Leftrightarrow y = x+1$$

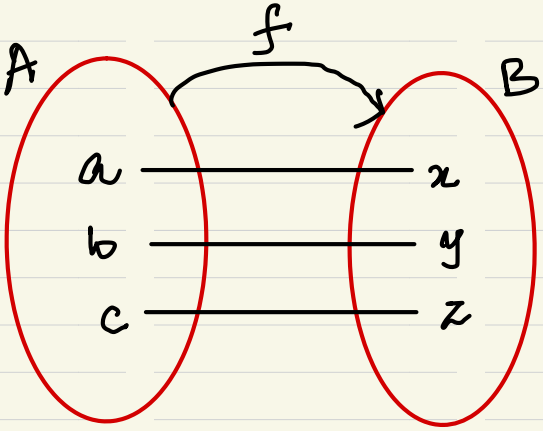
$$\Leftrightarrow x = y-1.$$

This means that for any $y \in \mathbb{R}$ there is $y-1 \in \mathbb{R}$ such that $f(y-1) = y$.

Hence f is onto.

$\#$

One-One-Onto function :-



$f : A \to B$ is an one-one-onto function.

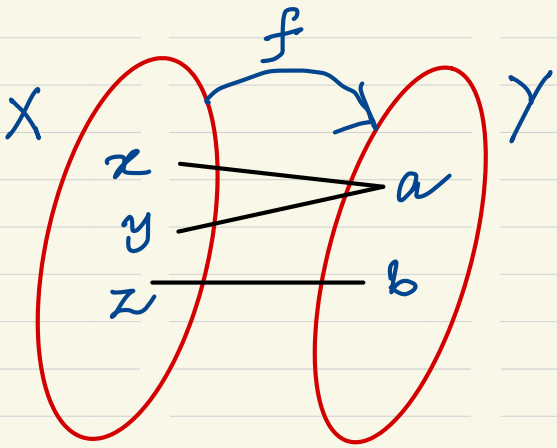
Note :- The inverse of a function f^{-1} exist, if f is One-One-Onto.

In above

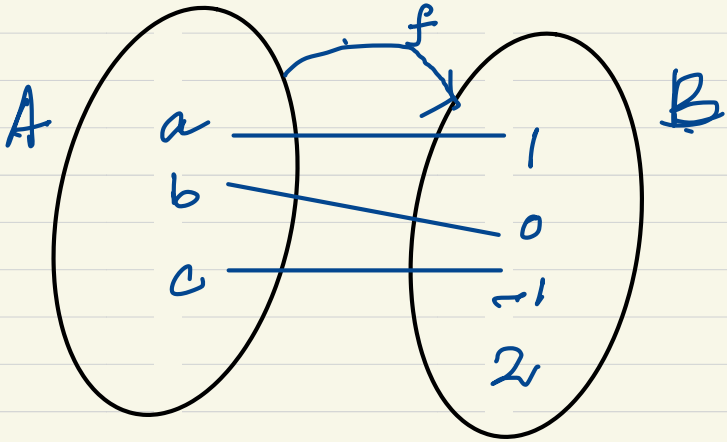
$f^{-1} : B \to A$ is also function.

RKBhuyan

Many-One-Onto:



Into function



Home Work :

1. One-One-into,
2. Many-one-into
3. Surjective mapping
4. Bijective "
5. Injective " .

ফলন। :: ভাগ –২

ফলনৰ শ্ৰেণীবিভাগ

Classification Of Function

Class No - 4

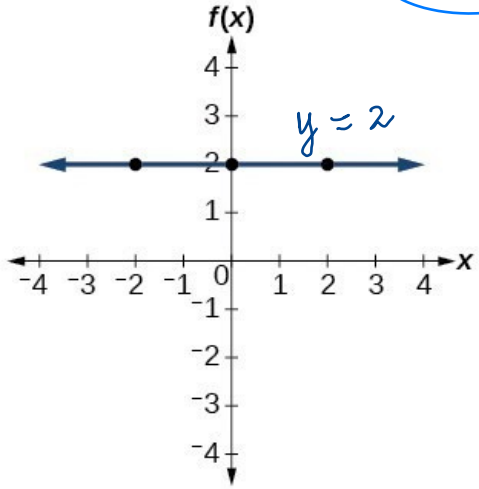
Classification of Function

1 Constant function: ধ্রুবক ফলন

$f(x) = k, \quad k \in R \; ; \; f: R \to R$

$f(5) = k, \; f(0) = k, \; f(\sqrt{2}) = k, \cdots$

$K = 2$



Table

x	f(x)
−2	2
0	2
2	2

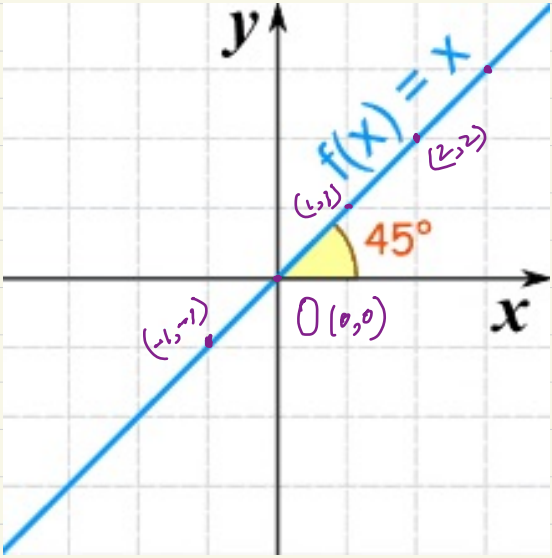
2. Identity Function: একৈকী ফলন

$$f(x) = x \quad : \quad f: R \to R$$

Table

Domain =Range=R

x	0	1	-1	2
$f(x)$	0	1	-1	2



3 (a) **Even Function** যুগ্ম ফলন

If a function is such that

$$f(-x) = f(x)$$

Then f is called Even function.

e.g. $f(x) = \cos x$, $f(x) = x^2$

(b) **Odd Function:** অযুগ্ম ফলন

If a function is such that

$$f(-x) = -f(x)$$

Then f is called odd function.

e.g. $f(x) = \sin x$, $f(x) = x^3$

4 Monotonic function:

একদিষ্ট ফলন

A function f is called monotonic iff it is either increasing or decreasing.

(a) Increasing Function:

বর্ধমান ফলন

Let $f(x)$ be a function and $x_1, x_2 \in D_f$.

If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

then f is increasing fu.n

e.g:- $f(x) = x^2$

Table

x	1	2	3	4	....
$f(x)$	1	4	9	16	....

4 b

Decreasing Function :

হ্রাসমান ফলন

Let $f(x)$ be a function and $x_1, x_2 \in D_f$.

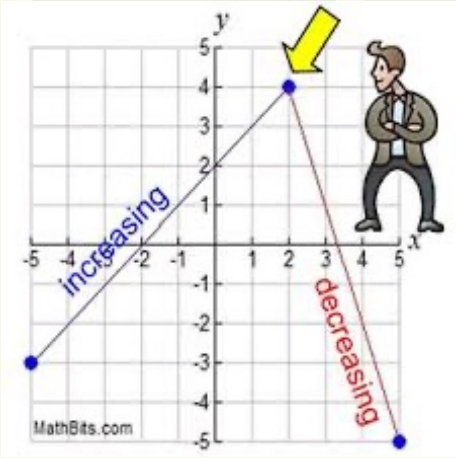
If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

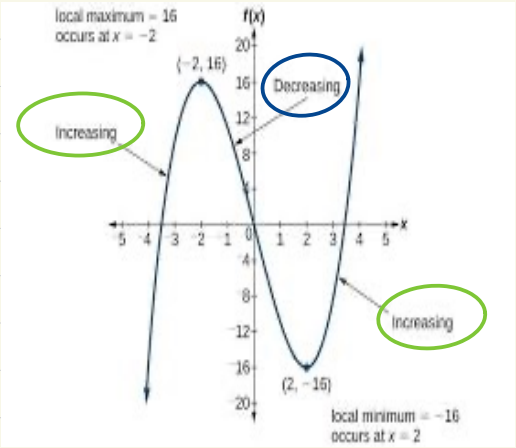
then f is decreasing fu.n

e.g :- $f(x) = 1/x$

Table

x	1	2	3	4	...	10	...
$f(x)$	1	.5	.33..	.25		.01	...





5 Absolute Value fu^{n}:

মাপাংক ফলন

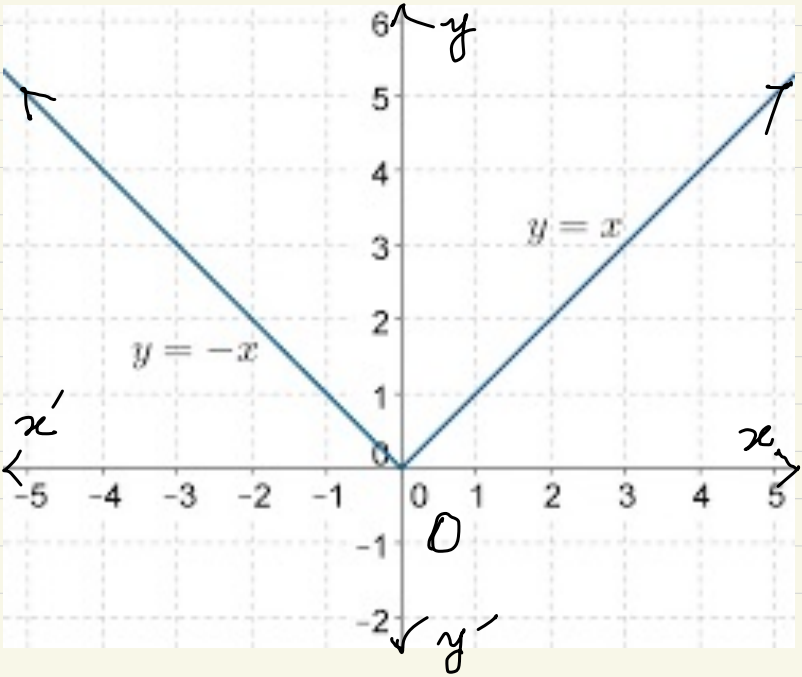
$$f(x) = |x| = \sqrt{x^2}$$

Domain = R. ; Range= [0, ∞)

বা $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Table

x	1	2	0		0	-1	-2
$f(x)$	1	2	0		0	1	2



6 Signum Function :-

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

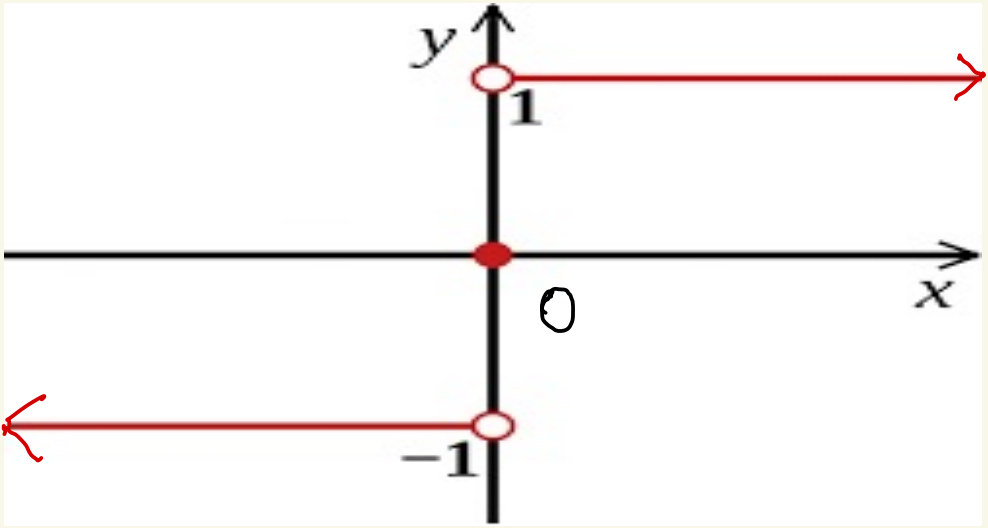
$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Domain=R

Range={ -1,0,1}

Note that f is not one-one

It is written by $\boxed{\text{Sgn. } x}$



ফলন। :: ভাগ -৩

ফলনৰ শ্ৰেণীবিভাগ

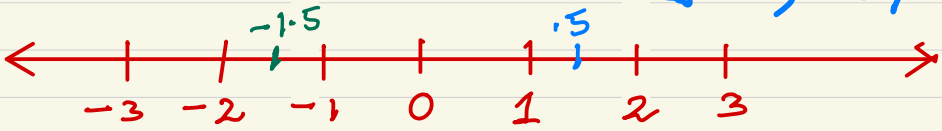
Classification Of Function

Class No - 5

7 Greatest Integer Function

$f(x) = [x]$, $[x] = x$, $x \in I$

$< x$, $x \notin I$

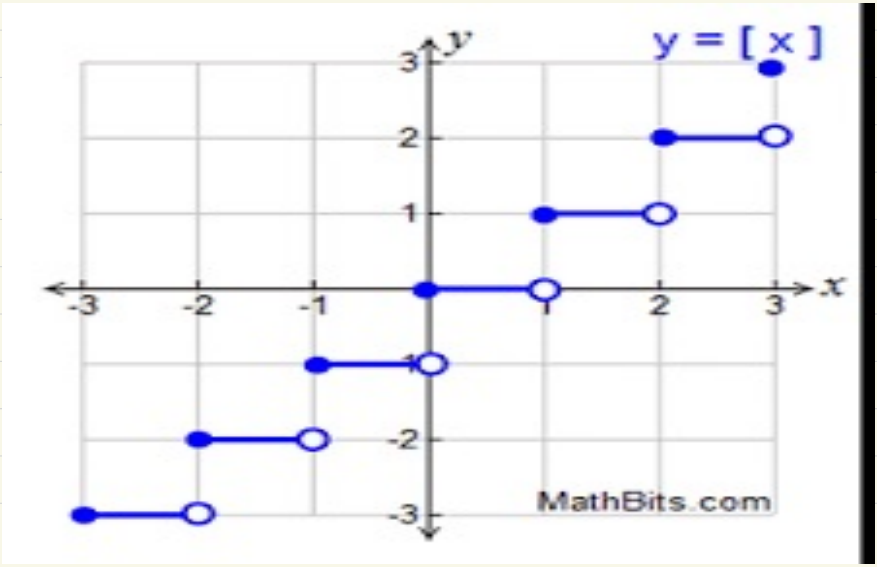


$[5] = 5$, $[0.5] = 0$, $[-1.5] = -2$

First left Integer

Table

x	[x]
0<=x<1	0
1<=x<2	1
2<=x<3	2



8 Parametric Function

প্রাচলিক ফলন

$$x = f(t), \quad y = g(t)$$

$t \rightarrow$ Parameter.

e.g. $x = a(\theta - \cos\theta), \quad y = 2a\sin\theta$

θ – parameter.

9 Implicit Function অন্তর্নিহিত ফলন

1. $x^2 + xy + y^2 = a^2, \quad f(x,y) = a$

2. $y = \cos(x+y)$.

10 Periodic Function পর্যাবৃত্ত ফলন

$$f(x) = f(x+\lambda) \quad , \quad \lambda > 0$$

e.g. $\cos x = \cos(2\pi + x)$

$$\text{Period} = 2\pi$$

11 Polynomial Function

বহুপদীয় ফলন

eg $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots$

$$y = ax^2 + bx + c$$

$$y = 2x^2 - 5x + 2$$